



AP[®] Calculus BC 2011 Scoring Guidelines

The College Board

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the College Board is composed of more than 5,700 schools, colleges, universities and other educational organizations. Each year, the College Board serves seven million students and their parents, 23,000 high schools, and 3,800 colleges through major programs and services in college readiness, college admission, guidance, assessment, financial aid and enrollment. Among its widely recognized programs are the SAT[®], the PSAT/NMSQT[®], the Advanced Placement Program[®] (AP[®]), SpringBoard[®] and ACCUPLACER[®]. The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities and concerns.

© 2011 The College Board. College Board, ACCUPLACER, Advanced Placement Program, AP, AP Central, SAT, SpringBoard and the acorn logo are registered trademarks of the College Board. Admitted Class Evaluation Service is a trademark owned by the College Board. PSAT/NMSQT is a registered trademark of the College Board and National Merit Scholarship Corporation. All other products and services may be trademarks of their respective owners. Permission to use copyrighted College Board materials may be requested online at: www.collegeboard.com/inquiry/cbpermit.html.

Visit the College Board on the Web: www.collegeboard.org.

AP Central is the official online home for the AP Program: apcentral.collegeboard.com.

AP[®] CALCULUS BC
2011 SCORING GUIDELINES

Question 1

At time t , a particle moving in the xy -plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

- Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.
- Find the slope of the line tangent to the path of the particle at time $t = 3$.
- Find the position of the particle at time $t = 3$.
- Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

(a) Speed = $\sqrt{(x'(3))^2 + (y'(3))^2} = 13.006$ or 13.007

Acceleration = $\langle x''(3), y''(3) \rangle$
 $= \langle 4, -5.466 \rangle$ or $\langle 4, -5.467 \rangle$

2 : $\begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$

(b) Slope = $\frac{y'(3)}{x'(3)} = 0.031$ or 0.032

1 : answer

(c) $x(3) = 0 + \int_0^3 \frac{dx}{dt} dt = 21$

$y(3) = -4 + \int_0^3 \frac{dy}{dt} dt = -3.226$

At time $t = 3$, the particle is at position $(21, -3.226)$.

4 : $\begin{cases} 2 : x\text{-coordinate} \\ 1 : \text{integral} \\ 1 : \text{answer} \\ 2 : y\text{-coordinate} \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) Distance = $\int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = 21.091$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

AP[®] CALCULUS BC
2011 SCORING GUIDELINES

Question 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \leq t \leq 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of $\frac{1}{10} \int_0^{10} H(t) dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10} \int_0^{10} H(t) dt$.
- (c) Evaluate $\int_0^{10} H'(t) dt$. Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

(a)
$$H'(3.5) \approx \frac{H(5) - H(2)}{5 - 2}$$

$$= \frac{52 - 60}{3} = -2.666 \text{ or } -2.667 \text{ degrees Celsius per minute}$$

1 : answer

(b) $\frac{1}{10} \int_0^{10} H(t) dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10} \int_0^{10} H(t) dt \approx \frac{1}{10} \left(2 \cdot \frac{66 + 60}{2} + 3 \cdot \frac{60 + 52}{2} + 4 \cdot \frac{52 + 44}{2} + 1 \cdot \frac{44 + 43}{2} \right)$$

$$= 52.95$$

3 : $\begin{cases} 1 : \text{meaning of expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{estimate} \end{cases}$

(c) $\int_0^{10} H'(t) dt = H(10) - H(0) = 43 - 66 = -23$
 The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.

2 : $\begin{cases} 1 : \text{value of integral} \\ 1 : \text{meaning of expression} \end{cases}$

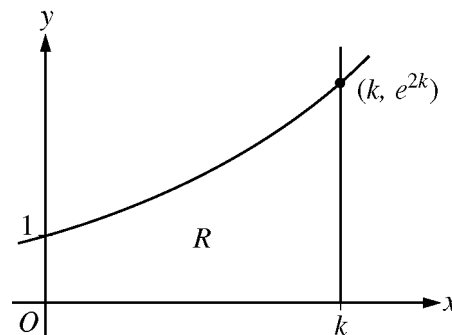
(d) $B(10) = 100 + \int_0^{10} B'(t) dt = 34.18275$; $H(10) - B(10) = 8.817$
 The biscuits are 8.817 degrees Celsius cooler than the tea.

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } B(0) = 100 \\ 1 : \text{answer} \end{cases}$

AP[®] CALCULUS BC
2011 SCORING GUIDELINES

Question 3

Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f , the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure above.



- (a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of R in terms of k .
- (b) The region R is rotated about the x -axis to form a solid. Find the volume, V , of the solid in terms of k .
- (c) The volume V , found in part (b), changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dV}{dt}$ when $k = \frac{1}{2}$.

(a) $f'(x) = 2e^{2x}$

$$\text{Perimeter} = 1 + k + e^{2k} + \int_0^k \sqrt{1 + (2e^{2x})^2} dx$$

$$3 : \begin{cases} 1 : f'(x) \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

(b) $\text{Volume} = \pi \int_0^k (e^{2x})^2 dx = \pi \int_0^k e^{4x} dx = \frac{\pi}{4} e^{4x} \Big|_{x=0}^{x=k} = \frac{\pi}{4} e^{4k} - \frac{\pi}{4}$

$$4 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

(c) $\frac{dV}{dt} = \pi e^{4k} \frac{dk}{dt}$

$$\text{When } k = \frac{1}{2}, \frac{dV}{dt} = \pi e^2 \cdot \frac{1}{3}.$$

$$2 : \begin{cases} 1 : \text{applies chain rule} \\ 1 : \text{answer} \end{cases}$$

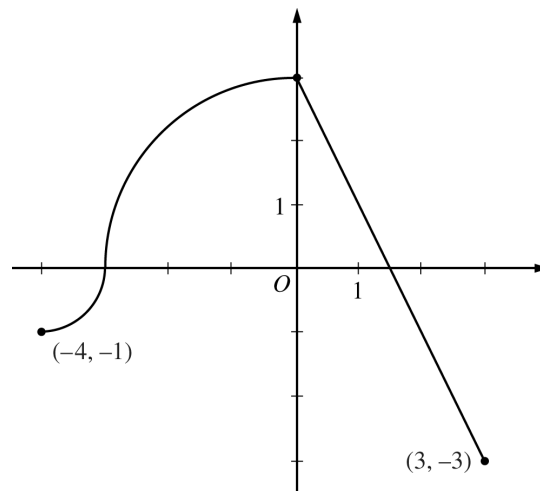
AP[®] CALCULUS BC
2011 SCORING GUIDELINES

Question 4

The continuous function f is defined on the interval $-4 \leq x \leq 3$.
 The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

Let $g(x) = 2x + \int_0^x f(t) \, dt$.

- (a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.
- (b) Determine the x -coordinate of the point at which g has an absolute maximum on the interval $-4 \leq x \leq 3$.
 Justify your answer.
- (c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.
- (d) Find the average rate of change of f on the interval $-4 \leq x \leq 3$. There is no point c , $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.



Graph of f

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t) \, dt = -6 - \frac{9\pi}{4}$
 $g'(x) = 2 + f(x)$
 $g'(-3) = 2 + f(-3) = 2$

3 : $\begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$

(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$.
 $g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$.
 Therefore g has an absolute maximum at $x = \frac{5}{2}$.

3 : $\begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$

(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.

1 : answer with reason

(d) The average rate of change of f on the interval $-4 \leq x \leq 3$ is
 $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$.

2 : $\begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$

To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.

AP[®] CALCULUS BC
2011 SCORING GUIDELINES

Question 5

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

- (a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).
- (b) Find $\frac{d^2W}{dt^2}$ in terms of W . Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.
- (c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

(a) $\left. \frac{dW}{dt} \right|_{t=0} = \frac{1}{25}(W(0) - 300) = \frac{1}{25}(1400 - 300) = 44$

The tangent line is $y = 1400 + 44t$.

$$W\left(\frac{1}{4}\right) \approx 1400 + 44\left(\frac{1}{4}\right) = 1411 \text{ tons}$$

$$2 : \begin{cases} 1 : \frac{dW}{dt} \text{ at } t = 0 \\ 1 : \text{answer} \end{cases}$$

(b) $\frac{d^2W}{dt^2} = \frac{1}{25} \frac{dW}{dt} = \frac{1}{625}(W - 300)$ and $W \geq 1400$

Therefore $\frac{d^2W}{dt^2} > 0$ on the interval $0 \leq t \leq \frac{1}{4}$.

The answer in part (a) is an underestimate.

$$2 : \begin{cases} 1 : \frac{d^2W}{dt^2} \\ 1 : \text{answer with reason} \end{cases}$$

(c) $\frac{dW}{dt} = \frac{1}{25}(W - 300)$

$$\int \frac{1}{W - 300} dW = \int \frac{1}{25} dt$$

$$\ln|W - 300| = \frac{1}{25}t + C$$

$$\ln(1400 - 300) = \frac{1}{25}(0) + C \Rightarrow \ln(1100) = C$$

$$W - 300 = 1100e^{\frac{1}{25}t}$$

$$W(t) = 300 + 1100e^{\frac{1}{25}t}, \quad 0 \leq t \leq 20$$

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } W \end{cases}$$

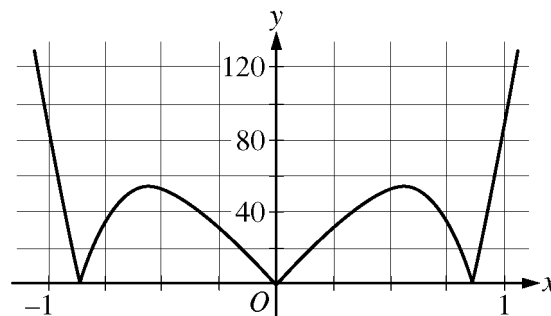
Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

AP[®] CALCULUS BC
2011 SCORING GUIDELINES

Question 6

Let $f(x) = \sin(x^2) + \cos x$. The graph of $y = |f^{(5)}(x)|$ is shown above.



Graph of $y = |f^{(5)}(x)|$

- (a) Write the first four nonzero terms of the Taylor series for $\sin x$ about $x = 0$, and write the first four nonzero terms of the Taylor series for $\sin(x^2)$ about $x = 0$.
- (b) Write the first four nonzero terms of the Taylor series for $\cos x$ about $x = 0$. Use this series and the series for $\sin(x^2)$, found in part (a), to write the first four nonzero terms of the Taylor series for f about $x = 0$.
- (c) Find the value of $f^{(6)}(0)$.
- (d) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Using information from the graph of $y = |f^{(5)}(x)|$ shown above, show that $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}$.

(a) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$
 $\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$

3 : $\begin{cases} 1 : \text{series for } \sin x \\ 2 : \text{series for } \sin(x^2) \end{cases}$

(b) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
 $f(x) = 1 + \frac{x^2}{2} + \frac{x^4}{4!} - \frac{121x^6}{6!} + \dots$

3 : $\begin{cases} 1 : \text{series for } \cos x \\ 2 : \text{series for } f(x) \end{cases}$

(c) $\frac{f^{(6)}(0)}{6!}$ is the coefficient of x^6 in the Taylor series for f about $x = 0$. Therefore $f^{(6)}(0) = -121$.

1 : answer

(d) The graph of $y = |f^{(5)}(x)|$ indicates that $\max_{0 \leq x \leq \frac{1}{4}} |f^{(5)}(x)| < 40$.

Therefore

$$\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| \leq \frac{\max_{0 \leq x \leq \frac{1}{4}} |f^{(5)}(x)|}{5!} \cdot \left(\frac{1}{4}\right)^5 < \frac{40}{120 \cdot 4^5} = \frac{1}{3072} < \frac{1}{3000}.$$

2 : $\begin{cases} 1 : \text{form of the error bound} \\ 1 : \text{analysis} \end{cases}$